

# Geometric image rectification: A review of most commonly used calibration patterns

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**Abstract-** Image is used in several areas such as mobile robots, intelligent vehicles, surveillance and so on. In many such applications which involve accurate quantitative measurements, image rectification is a crucial step. In this paper, we present a review of some of the most frequently used targets for camera calibration or image rectification and the different techniques to extract their coordinates in the image. We report at the end some of existence methods to evaluate the accuracy of target detection and the performance of calibration procedure.

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**Keywords:** Camera calibration; Image rectification; Calibration pattern; Lens distortion.

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## I. INTRODUCTION

Generally, the geometric deformation in the image is caused by radial and tangential distortions. Indeed, 3D Straight lines are projected as 2D curved lines in the image because of these two kinds of distortion. Radial distortion is due to the optics used and the flawed curves of the lens elements. Tangential distortion is expressed by two components: decentering distortion and affine deformation [1]. The misalignment of the lens components along the optical axis gives rise to the decentering distortion. The affine deformation is due to non-orthogonality of the image axes and dimensions of the pixel element, usually non- square. In usual photogrammetry and computer vision practices, commonly only radial distortion is considered because of its more significant influence on the image geometry. This component is relatively high for the fisheye lens. Therefore, other projection models were developed [2].

Camera calibration is a process whereby the geometric camera parameters are determined. It consists in the estimation of the projection model that represents the relationship between the target coordinates space and their projection in the image. It maps a distorted image into a perfect image that obeys the rectilinear model. Existing camera calibration techniques can be classified into two methods. The first technique is the traditional calibration where the calibration is performed by 2D or 3D coordinates of some reference targets, called calibration object [3]. The second technique, called self-calibration, is performed by using image information only [4]

This paper presents a detailed review of some of the most frequently used targets and the different techniques for image data extraction. Commonly, the different patterns used in the calibration procedure are designed to ensure that targets localization in the image is accurate using their properties such as color and geometry to improve accuracy.

The following Section describes the classical calibration procedure, presenting the rectilinear model, followed by the application of nonlinear optimization for solving parameters. Section 3 presents targets of calibration objects used to calibrate cameras. Section 4 studies different methods of pattern data extraction. Section 5 describes the

evaluation technique of detection accuracy and image rectification. Finally, the conclusion and discussion are presented in Section 6.

## II. CAMERA CALIBRATION AND PROJECTION MODEL

Camera calibration consists in the estimation of projection model that represents the relationship between the target coordinates  $(X, Y, Z)$  in the space and their projection in the image  $(u_p, v_p)$ . The rectilinear model is the most frequently used model in the calibration procedure [5]. It is based on the collinearity principle. The equation of this model is given by:

$$\begin{pmatrix} ku_p \\ kv_p \\ k \end{pmatrix} = M \cdot \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \quad (1)$$

where  $M$  is the projection matrix that describes the intrinsic and extrinsic parameters of the camera.

The rectilinear model is just an approximation referring to a projection in an ideal imaging system. A better suited model must take into account the displacement caused by radial and tangential distortions. Equation (2) models the application of the correction:

$$\begin{cases} u_d = u_p + \Delta u \\ v_d = v_p + \Delta v \end{cases} \quad (2)$$

where:

- $(u_p, v_p)$ : the undistorted or ideal image coordinates,
- $(u_d, v_d)$ : the distorted or true image coordinates
- $(\Delta u, \Delta v)$ : the relative corrections to the undistorted coordinates, giving the true image coordinates in the  $u$  and  $v$  directions.

The estimation of the camera parameters requires the application of an iterative algorithm to the residuals of  $N$  control points or targets between their observed values  $(U_{di}, V_{di})$  and the corresponding values  $(u_{di}, v_{di})$ , computed according to Equation (2). The following equation has to be minimized:

$$F = \sum_{i=1}^N (U_{di} - u_{di})^2 + (V_{di} - v_{di})^2 \quad (3)$$

The estimation of the distortion component allows rectifying geometrically the image. Indeed, the calibration procedure model the local transformations in the  $u$  and  $v$  directions for an image in order to eliminate the effect of lens distortion and transform a distorted image into a true rectilinear image.

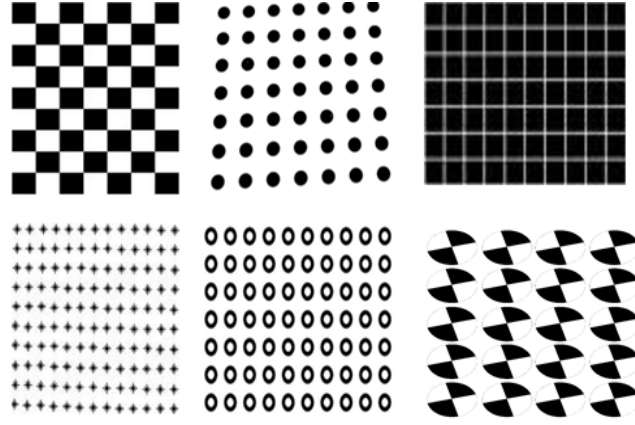
As a first step in the computation of the distortion components, the correction must be estimated at different targets. Then, the transformation is generalized to all pixels by using interpolation function [6].

## III. CALIBRATION PATTERN

A sufficient number of well-known targets is required to perform accurate camera calibration and image rectification. Many calibration objects have been proposed in the literature and can be classified in two main kinds: traditional calibration pattern and Structured-light calibration pattern.

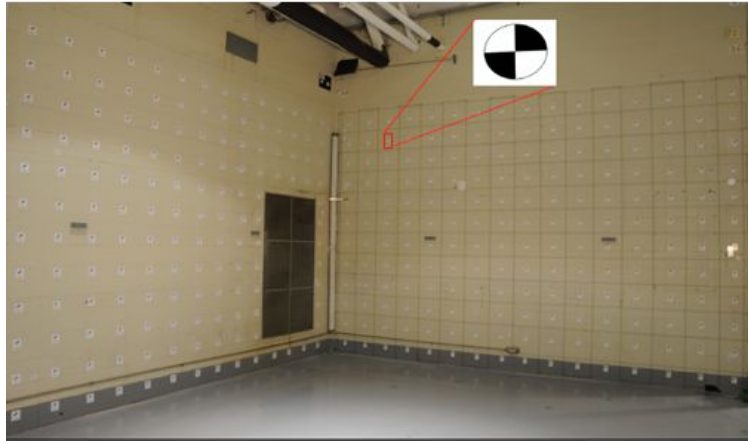
### A. Traditional calibration pattern

Traditional calibration 2D pattern is composed of a set of targets that may be points (node of intersection of horizontal and vertical, chessboard or square grid corners, center of circles or ellipses) [7] or other geometric forms such as straight lines [8], or spheres [9]. Figure 1 shows the most frequently used patterns which are simply printed on a standard commercial laser printer.



**Figure 1.** Traditional 2D pattern for camera calibration

Another kind of traditional calibration object is the 3D pattern (set of non-coplanar targets) which is very expensive to build and quite cumbersome to handle [3].



**Figure 2.** Example of 3D calibration pattern

In the case of digital aerial cameras, the calibration is generally performed by using a large room with hundreds of precisely surveyed targets in it [10]. These targets must be located far enough from the camera to simulate infinity and must be positioned at different distances from the camera to get a 3D effect that would cover the camera's entire field of view (FOV). Consequently, the relatively large distance from the camera and the distortion significantly contribute to the inaccuracy location of the targets in the image. The increase in their sizes (Figure 3) solves this problem. But, the effort of construction, installation and custom field becomes intense.

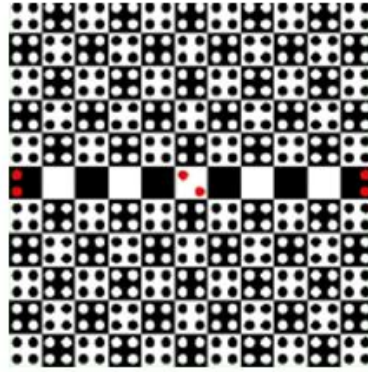


**Figure 3.** Example of target for aerial camera calibration

The fabrication of the pattern is an important challenge in zoom-lens camera calibration process. Particularly, as the magnification of the zoom system increases, the image size of the pattern becomes larger and thus the number of

targets within the FOV can become lower than that of control points needed to carry out the computation of parameters. Consequently, at large effective focal lengths for imaging at infinity, the target must be posed several meters away from the zoom camera, which requires a long calibration laboratory space or range.

The proposed solution in the literature is to develop a calibration object with two different patterns (example squares and circles in Figure 4). One pattern is used to calibrate high zoom level and the other to calibrate the low zoom level [11].

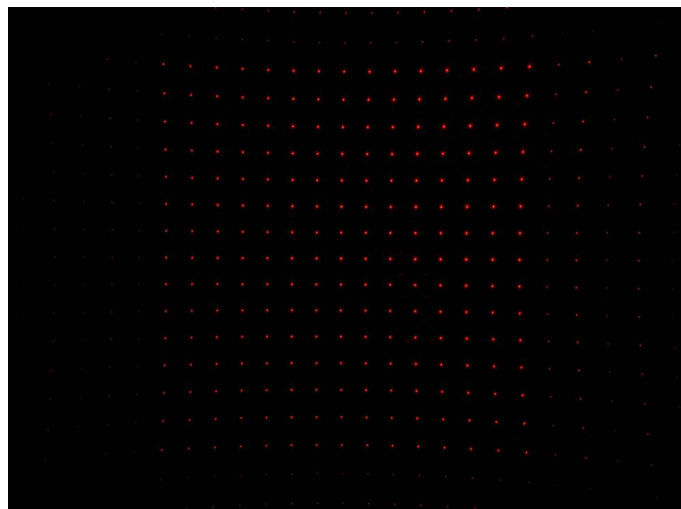


**Figure 4.** *A special pattern for zoom camera calibration*

Panoramic lens is an optical system that provides a wide field-of-view (FOV) of about 180 deg. To calibrate the panoramic cameras, many works have proposed the use of the chessboard or the square grid corners and the acquisition of multiple images to cover the entire field of view [12]. Other works have used a special calibration pattern adapted to the geometry of the sensor [13]. Several proposed methods are based on the projection of lines in the image. The main advantage is that a special pattern is not required [14].

### **B. Structured-light calibration pattern**

Structured light laser pattern can be easily obtained by using diffractive optical elements (DOEs). New setup using DOEs has recently been proposed in order to get a stable and compact calibration object [15]. The setup is based on two commercially available DOEs in a crossed configuration which generates a robust and accurate virtual calibration pattern suitable for different kinds of cameras.



**Figure 5.** *DOEs calibration grid*

#### IV. PATTERN DATA EXTRACTION

Accurate pattern detection is a fundamental step to establish a correspondence between targets in the object space and their projection in the image. Indeed, the calibration accuracy depends on the way they are measured. Several location methods have been proposed to achieve pattern data extraction using their properties such as color and geometry. They were sometimes complex and have provided subpixel accuracy.

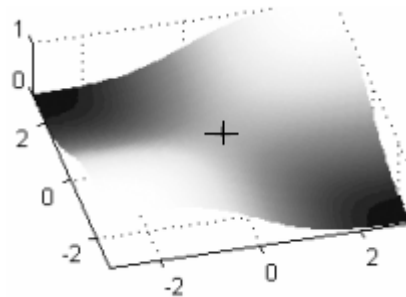
Existing extraction techniques can be classified into two methods: one based on the criterion of intensity and the other based on edge detection.

##### A. Subpixel detection based on intensity computation

Patterns with black and white squares or circles are most widely used because the easy subpixel detection algorithm for targets (corners of the chessboard or center of the circles) with high precision [16].

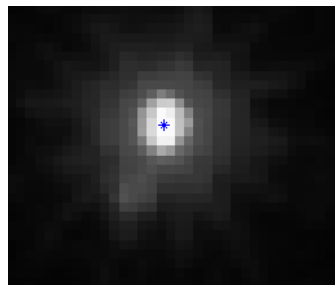
Traditional algorithm for detecting corners estimates firstly their pixel locations by standard corner detectors such as Harris. Then the subpixel positions can be determined using certain fitting procedures within a neighbouring area.

An illustration of these techniques was proposed by Reference [17]. Their algorithm detects the pixel position of the corner using Hessian matrix. Then a second order Taylor polynomial allows describing the local intensity profile around the corner. The subpixel position of the corner can be estimated by calculating the saddle point of this profile.



**Figure 6.** 3D intensity profile around X-corner

DOEs produces light spots on a black background. The control point coordinates are easily estimated with subpixel accuracy using an algorithm in gray scale mode [15].



**Figure 7.** Centroid detection of a DOES target

##### B. Subpixel detection based on edge estimation

The edge pixels of targets can be determined using any suitable edge detect technique. Hough transform can be used to detect shapes, such as circles and lines with accuracy. Then, the targets (centers of circles or lines intersection) can be estimated with subpixel accuracy by applying a least squares solution [3].



**Figure 8.** Edge detection of a target

## V. Accuracy evaluation and image rectification

Given the measurement error  $(e_u, e_v)$  for the target, Equation (2) can be written in function of observed values  $(U_d, V_d)$  as follows:

$$\begin{cases} U_d + e_u = u_p + \Delta u \\ V_d + e_v = v_p + \Delta v \end{cases} \quad (4)$$

In the method of nonlinear global modeling [3], the function E (Equation (5)) is designed to minimize the errors for N well distributed targets.

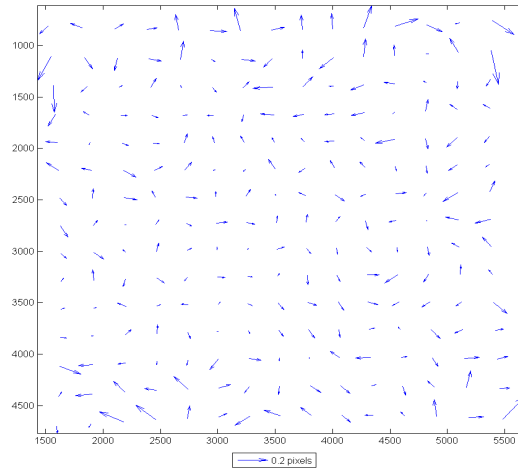
$$E = \sum_{i=1}^n (e_{ui} + e_{vi})^2 \quad (5)$$

The computation of distortion components allows converting a distorted image into a rectilinear image. There are two steps in the operation. First, the position of each target is adjusted while eliminating the distortion. Then the transformation is generalized to all pixels by interpolation techniques [6].

In many works in the literature, the evaluation of the subpixel detection technique is based on the root mean square error (RMSE), being:

$$RMSE = \sqrt{\frac{\sum_{i=1}^N (e_{ui} + e_{vi})^2}{N}} \quad (6)$$

Several other works have evaluated the detection performance by checking the accuracy of 3D reconstruction using images taken in varying orientations [6].



**Figure 9.** RMSEs computed by minimizing Equation (5) (in pixels)

The comparison of the two approaches, calibration using structured-light and traditional patterns, was performed. The experimental results showed that calibration technique using structured-light is more efficient [2].

## VI. CONCLUSION

In this paper, a review of the most commonly used patterns for camera calibration and image rectification have been presented. A sufficient number of well-distributed and known targets are required to perform accurate calibration.

The choice of the detection technique in the image depends on the pattern properties such as his color and his geometry. Two of existing methods for detection accuracy have been discussed.

A good pattern must provide a sufficient number of accurate targets to fulfill the entire FOV. Generally, the camera type, the required accuracy and the nature of the application are used to choose the number of targets and images suitable to carry out the camera calibration and the image rectification.

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